Appendix 3

The appendix includes:

- 3.1 The lake model dynamics: A detailed description of the lake model dynamics.
- 3.2 Objective functions for the lake problem: A detailed description of the objective functions used to evaluate candidate strategies.
- 3.3 Robustness metric description: A mathematical description of the robustness metric.

3.1 The lake model dynamics

The lake can exist in two states: oligotrophic or eutrophic. In the oligotrophic state, the lake has low concentrations of phosphorus with clear water. In the eutrophic states, the phosphorus concentration is high and algae can bloom. In the eutrophic state, the lake is assumed to be unable to support fisheries, or tourism, and also to be severely degraded in aesthetic value. It is also much harder to revert the lake from the eutrophic to the oligotrophic state in a short time by reducing pollution alone. Therefore, the transition to eutrophic state has multiple disadvantages, besides loss of economic activity.

Depending upon the ease with which a lake in eutrophic state can be brought back to its oligotrophic state, lakes can be classified as reversible, hysteretic, or irreversible. Irreversible lakes are most vulnerable since it is impossible to bring them back to an oligotrophic stage by reducing phosphorus concentrations alone once they exceed a threshold. In this study, the parameters of the lake model are such that the lake is irreversible and therefore represents an ecosystem with two possible states. Once the lake turns eutrophic, it is not possible to return it to an oligotrophic state by reducing phosphorus inputs alone. In reality, these conditions are most likely to occur in shallow lakes, lakes in phosphorus rich regions, or lakes that have received extreme phosphorus inputs for an extended period of time.

In the simple model, the parameter b determines whether the lake is reversible, hysteretic or irreversible for a given value of the recycling parameter q. Higher values of b suggest a lake that has a high capacity to remove pollution and vice-versa. If recycling occurs, the concentration of phosphorus in the lake increases suddenly over a period of time, the rate of this change is governed by the recycling parameter q. Higher values of q correspond to fast transitions and vice-versa. We adopt this formulation from the pioneering study by Carpenter et al. (1999). Carpenter et al. (1999) also provides a very careful and much more detailed description for the lake (model) system. Table 1 lists the parameter values for the lake model used in our study.

3.2 Objective functions for the lake problem

We begin with a widely used objective in the analysis of the lake problem - the expectation of discounted net present value of utility (O_1) given by,

$$O_1 = \frac{1}{N}\sum_{i=1}^{N} V_i \text{ , where,} \tag{3.1}$$

$$V_i = \sum_{t=0}^{T} \delta^t u_{t,i} \text{ , and,} \tag{3.2}$$

$$u_{t,i} = \alpha a_{t,i} - \beta X_{t,i}^2 . \tag{3.3}$$

In these equations, V_i (to be maximized) is the discounted net present value of utility for the i^{th} SOW, $u_{t,i}$ is the utility, $a_{t,i}$ is the allowed anthropogenic pollution and $X_{t,i}$ is the level of phosphorus in the lake at time step t and i^{th} SOW. The economic parameters α and β capture the willingness to pay for pollution and the compensation lake users are willing to accept to tolerate a given state of the lake respectively. α and β are fixed at 0.4 and 0.08 respectively following the analysis in Carpenter et al. (1999). For simplicity, we neglect the considerable uncertainty about the values for the discount rate and the economic parameters (Chichilnisky 1996, Dasgupta 2008). Note that the case for uncertain discounting was analyzed for the lake model by Ludwig et al. (2005). The discount factor, δ translates future to present utilities. The shortened term 'expected utility' is used to refer to this objective in the text and figures.

The time index, t, varies from 1 to T years (T = 100 years), and there are N SOWs. The SOWs are sampled from the lognormal distribution in Equation (1) and their total number (N) varies from 0 to 90000 based on the type of uncertainty being considered as described in the section on 'Uncertainty'. N is 0 for the deterministic case, 10000 for well-characterized uncertainty and 90000 for deep uncertainty. The allowed anthropogenic pollution flow a_t is only decision variable that controls the objective function. The stakeholder can change a_t only every 5 years. As a result, there are 20 planning periods across a planning horizon of 100 years and the optimization framework needs to identify the 20 values of a_t that satisfy selected stakeholders' objectives.

To contrast the strategy that maximizes the expected utility (O_1), we introduce additional objectives that represent stakeholders that more strongly focus on the long term environmental quality of the lake or are varied in their inter-temporal presence. Stakeholders often assess outcomes using a diverse set of objectives (Kasprzyk et al. 2009, McInerney et al. 2012, White et al. 2012, Herman et al. 2014). Farber et al. (2006) for example, argue that the linking of ecology and economics requires identification of ecosystem services that are likely to be in conflict. Our objective formulation is to a large part motivated by this assessment.

Identifying key objectives that represent diverse stakeholders is challenging and a potentially iterative process. Some of these objectives are a proxy for ecosystem services (recreation, fishery), while others serve as proxies for alternative perspectives with regard to valuing economic services (utility). This approach can be interpreted as representing the perspectives of five hypothetical stakeholder groups in the fictitious town. This resulted in the following objectives considered in our analysis:

1. Minimize the average level of phosphorus in the lake (O_2) – Admiraal et al. (2013) point out that the utility function is strongly biased towards anthropogenic services which is a key limitation in identifying ecosystem management strategies that adequately protect environmental values. Here, we introduce this objective to represent a regulatory

perspective related to an indicator of the health of the lake. This objective can be interpreted as one key concern of individuals that aim to preserve the lake as it is and therefore their sole goal is to reduce the levels of phosphorus in the lake. The objective function is,

$$O_2 = \frac{1}{NxT} \sum_{i=1}^{N} \sum_{t=1}^{T} X_{t,i} , \qquad (3.4)$$

where, $X_{t,i}$ is the phosphorus in the lake at time step, t and i^{th} SOW. This objective aims to minimize the average levels of phosphorus in the lake.

2. Maximize the expected utility of the present stakeholders (O_3) – This objective represents utility of the current stakeholders. The objective function is

$$O_3 = \frac{1}{N} \sum_{i=1}^{N} U_{1,i} , \qquad (3.5)$$

where, $U_{1,i}$ is the utility of the first year in the 100 year planning horizon for the i^{th} SOW and is to be maximized.

3. Maximize the expected utility of the future stakeholders (O_4) – The objective was motivated by the definition of sustainability adopted by past studies (Holling 1973, United Nations 1987, Cato 2009). These definitions represent the interest of present and future generations quite differently than the discounted expected utility framework. While discounting has been the classic approach to analyze inter-temporal trade-offs, several reports, even governmental decisions have been based on objectives that are not subject to discounting. One simple example is the design of flood defenses in the Netherlands that are subject to an acceptable level of risk (Jonkman 2013). Therefore, we explicitly model inter-temporal stakeholders in separate objective functions. To approximate this perspective, we choose two example stakeholder groups (i) current generation and (ii) generations in the far future. (Far here is represented as the second half of the planning horizon of the problem). This objective represents the utility of future stakeholders who exist in the last 50 years of the 100-year planning horizon. The objective function is

$$O_4 = \frac{1}{N} \sum_{i=1}^{N} U_{50-100,i} , \qquad (3.6)$$

where, $U_{50-100,i}$ is the sum of undiscounted utilities for the generations spanning years 50 to 100 in the 100-year planning horizon for the i^{th} SOW. This objective function is to be maximized.

4. Maximize reliability (O_5) – One of the goals of this study is to capture the behavior of multi-state ecosystems when some states are far less preferable to the stakeholder. The reliability objective seeks to ensure that the lake remains below critical pollution levels to avoid eutrophication. This objective also represents key concerns of stakeholders who either depend directly on the ecosystem services provided by the lake, or those who aim to maintain the ecosystem itself while being able to accept some levels of pollution. In

addition, this formulation approximates a common risk-based engineering metric that has been widely employed across many contexts (Hashimoto et al. 1982). Maximizing the reliability of avoiding a tipping point response captures the strong aversion to irreversible losses of key economic and ecosystem services. The objective is,

$$
\begin{aligned}
& O_5 = \frac{1}{NxT} \sum_{i=1}^{N} \sum_{t=1}^{T} \theta_{t,i} \\
& where, \theta_{t,i} = \begin{cases} 1 & if \quad X_{t,i} < X_{crit} \\ 0 & if \quad X_{t,i} \geq X_{crit} \end{cases} .
\end{aligned}
\tag{3.7}
$$

In these equations, $\theta_{t,i}$ is the reliability index which is 1 if the level of phosphorus in the lake ($X_{t,i}$) is below the specified critical threshold (X_{crit}) and 0 otherwise. The critical threshold is set at 0.5 based on the parameters of the lake model. X_{crit} is the minimum steady state pollution value at which the lake transitions from an oligotrophic to eutrophic state. A reliability of 1 represents a pollution strategy that successfully keeps the phosphorus levels in the lake below the specified critical thresholds across the entire planning horizon and across all SOWs. Table 3 lists the objectives used in this study.

3.3 Robustness metric description

We define performance requirements for key variables and a strategy that equal or exceeds these requirements across a range of uncertain scenarios is considered to be robust. An overall measure of robustness is thus defined as –

$$
\begin{aligned}
& \%Robust = \left(\frac{1}{N} \sum_{i=1}^{N} r_i \right) x100, \\
& where, r_i = \begin{cases} 1 & when\ P_{o,i} \geq requirement_O \ \forall \ o \\ 0 & otherwise \end{cases} .
\end{aligned}
\tag{3.8}
$$

Here, r_i is 1 if the strategy performs above all requirements otherwise 0, $P_{o,i}$ is the value of the o^{th} variable of interest under the i^{th} SOW, $requirement_o$ is the performance requirement for the o^{th} variable, and N is the total number of SOWs (10000 for well-characterized uncertainty and 90000 for deep uncertainty).

The performance requirements represent criteria that actual stakeholders may consider as performance levels that could not be compromised further. For example, stakeholders are likely to have a factor of safety associated with the critical phosphorus levels. Here, we fix that factor of safety at 0.75. Similarly, a high reliability of keeping the lake in the oligotrophic state is binding due to obvious economic and environmental consequences. Thus, the performance requirement on reliability was fixed at 99%. While maintaining the lake in an oligotrophic state is important, a minimum level of economic activity is also required. This level was set at 50% of the value of expected utility obtained in the optimal strategy for expected utility maximization (P2). Our proposed definition of robustness is illustrative and the MORDM framework is highly flexible in accommodating alternative definitions. The primary intent of our example is to emphasize that system performance requirements are themselves likely to be multi-objective, complex in their effects on filtering solutions, and should be carefully elicited in any real application of MORDM.

Note that our proposed definition of robustness spans multiple objectives and hence, prevents a stakeholder heavily biased towards one objective (say utility) from selecting strategies that favor their preferred objective. For the robustness index of a strategy to be high, all performance criteria need to be simultaneously satisfied across multiple SOWs. So, if a high performance requirement is selected for the utility function, strategies satisfying it may not satisfy the reliability or phosphorus requirements. Thus, it is likely that none of the strategies emerges as robust forcing stakeholders to revise their threshold specifications.

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