APPENDIX 1: DESCRIPTION OF THE MODEL USED IN THE SIMULATIONS.

The model has two components: an adaptive controller, and a dynamic system whose adaptive capacity depends on the magnitude of effort expended for adaptive control in the past.

Any one of several kinds of adaptive controllers could be used. Simulations reported here use a "proportional-integrated-derivative", or PID, controller (Anderson and Moore 1971).

Many kinds of dynamic systems could be modeled, such as the various social-ecological systems reviewed in the text. To keep the paper simple, we used a minimal one-dimensional dynamic system that tracks cumulative effort used for adaptive control, and relates this variable to adaptive capacity through the relationship shown in Figure 1.

First we will describe the discrete-time version of the model which was used for simulations.
The model (equations 1-6 below) consists of a PID controller (eqs. 1-4) coupled to a model for the accumulation and dissipation of stress (eq 5), and a model relating stress to adaptive capacity (eq. 6). The system state y adapts to follow a changing target, y* at a particular time. Deviations from the target are measured by x (eq. 1). The dynamics of y (eq. 2) depend on history (through the "a" term), adaptive adjustments (the "b" term), and disturbances (the "n" term). In PID control, the magnitude of the adaptive adjustment u (eq. 4, Fig. 2) is a weighted combination of the integrated past deviation from target (the "f " term; note from eq. 3 that q accumulates the integral), current (proportional) deviation from target (the "h_P" term), and the rate of change (derivative) of deviation from target (the "h_D" term). Stress accumulates in relation to the magnitude of control exerted, and decays over time (eq. 5). The hat-shaped function for A(S) is modeled using eq. 6. The steepness of the function and its symmetry are adjustable using the J parameters.

[1] $x_t = y_t - y_t^*$

[2] $y_t = ay_{t-1} + bA(S_{t-1})u_t + n_t$

[3] $q_t = mq_{t-1} + kx_{t-1}$

[4] $u_t = -fq_{t-1} - [h_P x_{t-1} + h_D(x_{t-1} - x_{t-2})]$

[5] $S_t = S^* + c_u u_t^2 + (1 - c_S)(S_{t-1} - S^*)$

[6] $A(S_t) = 0.5\{\tanh[J(S_t - s_1)] - \tanh[J(S_t - s_2)]\}$

Parameters used for simulations in the text are presented in Table A1, at the end of this Appendix.

The model could also be written in continuous time (eqs. 1' – 6' below). Because the continuous time model must be discretized for computer simulation, we worked directly with the discrete time equations. We present the continuous time version for completeness. In eq. 2', σ is a standard deviation and W is a Weiner stochastic process.

[1'] $x_t = y_t - y_t^*$

[2'] $$\frac{dy}{dt} = (a-1)y + bA(S)u + \sigma\frac{dW}{dt}$$

[3'] $$\frac{dq}{dt} = (m-1)q + kx$$

[4'] $$u = -fq - [h_P x + h_D \frac{dx}{dt}]$$

[5'] $$\frac{dS}{dt} = c_u u^2 - c_S (S - S^*)$$

[6'] $$A(S) = 0.5\{\tanh[J(S - s_1)] - \tanh[J(S - s_2)]\}$$

References

Anderson, B. and J. Moore. 1971. *Linear Optimal Control*. Prentice-Hall, Englewood Cliffs, New Jersey, USA.

Table A1. Definition of symbols for the discrete-time model and values used in the simulations shown in the main text.

Symbol	Definition	Nominal Value (if constant)
a	Autoregressive parameter for state variable (constant)	1
$A(S_t)$	Function for the adaptive capacity defined by eq. 6	-
b	Coefficient for the effect of the control (constant)	1
c_S	Contribution of past stress to current stress (constant)	0.1
c_u	Contribution of current control magnitude to stress (constant)	0.5
f	Contribution of integral deviation to the control (constant)	0.4
h_D	Contribution of the rate (difference) of deviation to the control (constant)	0.2
h_P	Contribution of proportional deviation to the control (constant)	0.6
J	Slope parameter for the adaptive capacity function (constant)	-
k	Weight of current deviation of the state variable from the target (constant)	1
m	Weight of accumulated past deviations of the state variable from the target (constant)	1
n_t	Disturbance to state variable dynamics at time t	-
q_t	Accumulated weighted deviations of the state variable from the target	-
S^*	Long-run average stress (constant)	see Figs 3, 4, 5
s_1	Lower inflection point for the adaptive capacity function (constant)	3
s_2	Upper inflection point for the adaptive capacity function (constant)	7
S_t	Stress at time t	-
tanh	Hyperbolic tangent function	-
u_t	Control magnitude at time t	-
x_t	Deviation of the state variable from the target at time t	-
y_t	State variable at time t	-
y_t^*	Target at time t	-