Computing *V(h)*:
In order to characterize the function *F(h)*, we first need to determine the expression of *V(h)* where the log of the habitat amount *h* is governed by the differential equation

$$dh = \alpha\left(h_b - h\right)dt + \sigma dz$$

We focus on the case of interest $h > h_e$; then,

$$V\left(h\right) = E\left(\int_0^{dt} s_s e^{-rs} ds + \int_{dt}^{+\infty} s_s e^{-rs} ds\right)$$
$$\Rightarrow V\left(h\right) = sdt + e^{-rdt} EV\left(h + dh\right)$$
$$\Rightarrow V\left(h\right) = sdt + (1 - dt)E(V(h) + V'(h)dh + \frac{1}{2}V''(h)dh^2) + o(dt)$$
$$\Rightarrow \frac{E\left(dh\right)^2}{dt}\frac{V''(h)}{2} + \frac{Edh}{dt}V'(h) - rV(h) + s + o(dt) = 0$$

When *dt* goes to *zero* in the equation above, we obtain the partial differential equation governing *V(h)*

$$\frac{\sigma^2}{2}V''(h) + \alpha\left(h_b - h\right)V'(h) - rV(h) + s = 0,\ \forall\ \text{h} > h_e$$

along with the boundary condition $V(h_e) = 0$. This boundary condition is said to be 'absorbing', when *h* hits h_e for the first time. In this case the caribou becomes extinct, due to a lack of habitat, and the stochastic process becomes irrelevant (one says it is terminated or "killed").

Computing *F(h)*:
Initially *h* is equal to $h_o > h^* > h_e$, *it then* evolves until it hits h^* at time

$$T \in \left[0, +\infty\right]$$

However, if *T* is infinite, this means that *h* never hits h^*. At any time before *T* i.e. for any $h > h^*$, *F(h)* satisfies the Bellmann's Equation

$$F\left(h\right) = \left(\rho + s\right)dt + e^{-rdt} EF(h + dh)$$

where *h* becomes *h+dh* during the infinitesimal time period *dt*. Thus, *F(h)* is governed by the following partial differential equation, that is

$$\frac{\sigma^2}{2}F''(h) + \alpha\left(h_a - h\right)F'(h) - rF(h) + \rho + s = 0,\ \forall\ \text{h} > \text{h}^*$$

along with the following Value Matching (VM) and Smooth Pasting (SP) boundary conditions

$$(VM): F(h^*) = V\left(h^*\right)$$
$$(SP): F'(h^*) = V'\left(h^*\right)$$

The first condition says that, when forestry is costly banned, the value of the forest stems from the existence of the caribou only, as described above. The second condition requires this to happen ‘smoothly’, as it can be shown that it would not be optimal to ban forestry at that time if that condition was not met.