

Research, part of a Special Feature on [Crossing Scales and Disciplines to Achieve Forest Sustainability](#)  
**A Real Options Approach to Forest-Management Decision Making to Protect Caribou under the Threat of Extinction**

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**ABSTRACT.** Uncertainty is a dominant feature of decision making in forestry and wildlife management. Aggravating this challenge is the irreversibility of some decisions, resulting in the loss of economic opportunities or the extirpation of wildlife populations. We adapted the real options approach from economic theory to develop a methodology to evaluate a resource management decision to stop timber harvesting when a woodland caribou (*Rangifer tarandus caribou*) population becomes threatened with extinction. In our study area of central Labrador, Canada, both caribou and timber harvesting are valued ecosystem services. By using a decision rule, which incorporates future developments, the real options approach provides a technique to incorporate ecological and social uncertainty into forest-management decision making. As a result, it reduces the risk of a forest manager making a decision with unwanted irreversible consequences or failing to make a decision that could avoid such unwanted consequences.

**Key Words:** *decision support; real options; forest planning; wildlife management; caribou; Labrador;*

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## INTRODUCTION

Managing for ecosystem services such as timber and wildlife is fraught with various forms of uncertainty. Sources of uncertainty include external disturbances such as wildfire, system responses to change such as ecological change from timber extraction, and the underlying structure of systems, given that our knowledge of how ecosystems respond to forest management is incomplete (Walters 1986). The irreversibility of forestry and wildlife decisions complicates the management of forest ecosystems and wildlife populations. For example, once old-growth forests have been cut, the option of preserving them is lost (Conrad 2000). Alternatively, a decision to stop harvesting would preclude the socioeconomic opportunity of timber extraction that may benefit a resource-dependent community. Trading off the socioeconomic risks of preservation and the ecological risks of timber harvesting is a fundamental challenge for resource-management decision making (RMDM). There is a need for methods to deal with the risk, uncertainty, and irreversibility of RMDM. To investigate this need, we adapted an approach called “real options,” which is used in economics to evaluate financial decision making (Dixit and Pindyck 1994).

The risk and uncertainty associated with management decisions are included in the formulation of real options problems (Dixit and Pindyck 1994). A real option is defined as “the value of being able to choose some characteristic (e.g., the timing) of a decision with irreversible consequences, which affects a real asset (as opposed to a financial asset)” (Saphores and Carr 2000). Under real options, problems are structured so that they can be solved by numerical methods. It has been applied in RMDM in areas such as species reintroduction (Bakshi and Saphores 2004), biodiversity (Kassar and Lasserre 2002), forest harvesting (Insley 2002), land-use decisions (Marwah and Zhao 2002), and climate change (Boyer et al. 2003). Real options uses a flexible approach to uncertainty by identifying its sources, developing future scenarios, and constructing decision rules (Boyer et al. 2003). It attempts to reduce risk by monitoring the implementation of its decisions and requiring decision making to be adaptive throughout the life cycle of a project. To gain the most benefit from a venture, be it a business venture or the management of a natural resource, the real options approach values future opportunities. For example, the real options method treats the loss of a wildlife species as if it were the loss of a future economic opportunity

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by recognizing that the species may have some future value (Kassar and Lasserre 2002).

The real options method explicitly accounts for uncertainty in the determination of an optimal decision in light of the stochasticity of an asset's value. For example, Brennan and Schwartz (1985) looked at the decision to open, close, and abandon a mine producing a resource whose price, and hence its value, was stochastic. Pindyck (2000) evaluated the timing of adopting an environmental policy when, because of climate change, there are uncertain costs and benefits to society. The real options approach also takes into account the irreversibility of resource management decisions by, e.g., evaluating the harvesting of old-growth forests in terms of their amenity value (Conrad 1997, Forsyth 2000), growth, and timber price (Clarke and Reed 1989, Reed and Clarke 1990). Marwah and Zhao (2002) examined the problem of irreversibility and uncertainty in land acquisition for wildlife conservation. Conversion of land from other uses, such as agriculture or forestry, may be irreversible, and the costs invested in preserving the land may be lost if that area becomes unsuitable for wildlife. Irreversibility infers unintended change, in which something of value is lost and must be considered when making risky decisions. Thus, the decision maker prefers to delay making a decision until more information about its possible consequences is available. However, by waiting, opportunities may deteriorate or be lost, such as vanishing chances to gain financial revenues or the decline in the population of a valued wildlife species. In the case of Marwah and Zhao (2002), they used real options methods to evaluate the effect of different decision-maker strategies for managing uncertainty in their determination of the optimal timing and amount of land to purchase for conservation. They found that the timing of land investments was highly dependent on how a decision maker synthesized existing and new information: A decision maker who actively preserves land and thereby learns about its costs and benefits to conservation objectives produces a more optimal solution than a passive decision maker who waits for better information.

In this analysis, we demonstrate how a decision maker would make use of the real options method to evaluate the trade-off between harvesting timber and maintaining sufficient old forest to support an endangered woodland caribou (*Rangifer tarandus caribou*) population in central Labrador, Canada.

We focus on the interaction of natural wildfire and timber exploitation, which increases the probability of local caribou extinction, and a community that forgoes timber revenues when forest harvesting is banned. To apply this methodology, the decision maker tracks the amount of caribou habitat and determines the optimal point to stop timber harvesting, given the uncertainty of future natural disturbances and the amount of habitat required to support a viable caribou population. The optimal stopping time is also informed by the social trade-off of maintaining the economic benefits to the community of timber harvesting and the conservation benefits of woodland caribou. The methodology presented in this paper is a component of a broader sustainable forest management project focused on central Labrador (Sturtevant et al. 2007).

The caribou population in central Labrador's Red Wine Mountains (RWM) declined from more than 700 animals in the 1980s to 151 in 1997 (Schaefer 1999). In 2002, the population was listed as "threatened" under both the Endangered Species Act of Newfoundland and Labrador and under the Canadian federal Species at Risk Act (Schmelzer et al. 2004). Caribou are an integral part of the communities of central Labrador. Caribou meat has historically constituted a large portion of the diet of the people in central Labrador. The Innu, the local first nation, have a strong spiritual and cultural connection with the caribou (Armitage 1992, Schmelzer 2004). As a conservation measure, nonsubsistence hunting of the RWM herd stopped in 1972, and subsistence hunting stopped in 2002 (Schmelzer 2004). The George River herd, one of the world's largest at 600,000 to 800,000 animals (Couturier et al. 1996), is considered healthy and is legally hunted. However, the ranges of the George River and RWM herds overlap, and the animals are nearly indistinguishable. When the two herds mix, the RWM caribou are frequently mistaken for the George River animals and shot, contributing to their decline (Schaefer 1999, Schaefer et al. 2001, Schmelzer 2004).

Currently, there is a proposal to substantially expand forest harvesting, some of which is planned within the historic range of the RWM caribou herd (Department of Forest Resources and Agrifoods 2003). Expanded harvesting is expected to provide economic benefits to local communities. Past timber exploitation, which was recent and small scale, has not been implicated in the decline of the RWM caribou herd, which was caused mainly by historical

hunting (Schmelzer 2004). Notwithstanding, commercial forestry has had a negative impact on caribou population dynamics and behavior across the boreal forest (Chubbs et al. 1993, Seip and Cichowski 1996, James and Stuart-Smith 2000, Schaefer 2003), and there is concern that expanded commercial forestry and the associated human activity in the range of the RWM caribou may further compromise their viability. As a measure to protect the endangered RWM herd, the local resource management plan has set aside large areas free from forest harvesting. However, it is uncertain if this is enough to ensure the caribou's survival.

Sedentary caribou (Bergerud 1988) such as the RWM herd exist in low numbers. Although lichen is a key food source for caribou, its availability is not considered to be the limiting factor in the persistence of caribou populations. Instead, the limiting factor is the distribution of winter and summer habitat (Seip and Cichowski 1996). Caribou need to be spatially separated from their predators and require large tracts of undisturbed forest. If caribou are confined to small areas, predators can find them more easily (Seip 1991). Smith et al. (2000) found that, as forest harvesting progressed in a landscape occupied by caribou, their daily movement rates and winter range size decreased, and they avoided recently fragmented areas. Conservation research has shown that the decline of a species is often associated with a degradation of its range (Channell and Lomolino 2000). According to Rempel et al. (1997), forest harvesting also causes a shift in the forest age structure to a higher occurrence of young forest, which is more favorable to other ungulates such as moose (*Alces alces*). With a larger prey base, the wolf (*Canis lupus*) population expands, preying on moose and the resident caribou (Bergerud and Elliot 1986, Seip 1992, Seip and Cichowski 1996). Wolves are the main nonhuman source of mortality for adult RWM caribou (Schaefer 1999). Recently, it has been observed that central Labrador's moose population has increased, whereas the number of RWM caribou has declined (Schaefer 1999). An additional concern related to expanded commercial forestry is increased road density. The efficiency of predators in a harvested landscape is facilitated by the increase in the number of roads and trails that result from forest harvesting (James and Stuart-Smith 2000). These same roads make it easier for poachers and legal hunters to gain access to the George River herd and the overlapping RWM caribou.

There are conflicting social and ecological risks for the Labrador study area. There is the ecological risk of the RWM caribou becoming extinct with expanded commercial forestry. As well, there is a risk that a socioeconomic opportunity will be lost if the decision to stop timber harvesting is made too soon. The timing of the decision to stop harvesting, before a critical minimum amount of caribou habitat is lost, is also dependent on the amount of certainty that the decision maker has about the viability of the caribou population under various levels of forestry activity. The decision maker needs to evaluate the certainty associated with how natural ecological dynamics may interact with timber harvesting and how it may undermine the amount of caribou habitat. In the social domain, the decision maker needs to consider what level of risk society is willing to accept given its interest in both a viable caribou population and the economic benefits that forestry brings to local communities.

In this study, we describe a real options methodology and how it deals with risk, uncertainty, and irreversibility. Also presented are background on data requirements and how parameters were calculated for the real options model. Caribou habitat is captured using a coarse habitat indicator. The real options methodology, the assumptions used, and the model are presented, followed by a discussion on the usefulness of this approach in RMDM, specifically in the Labrador study area.

## METHODS

### Study area

Our study area is defined as Labrador's District 19A. The Red Wine Mountains (RWM) caribou herd overlaps this area (Fig. 1). The local forest management plan outlines strategies to expand commercial forestry and to protect the resident woodland caribou habitat (Department of Forest Resources and Agrifoods 2003). The study area is approximately 2 million ha and is located in the lower section of the Churchill River Valley and the coastal plain surrounding Lake Melville (Fig. 1). Human impact, primarily in the form of roads, historic low levels of timber harvesting, and human-caused fires are confined mainly to the area on the north side of the Churchill River (Forsyth et al. 2003). The area has cool summers and cold winters and is the most heavily forested area of Labrador. Fire is infrequent and patchy, and the area's

disturbance regime is dominated by individual tree mortality. This results in a mixture of age and cover types and a multilayered canopy. The landscape is dominated by lakes, rivers, and wetlands, with forests of black spruce (*Picea mariana*) and balsam fir (*Abies balsamea*), open sphagnum forests, lichen woodlands, black spruce bogs, and stands of birch (*Betula papyrifera*), trembling aspen (*Populus tremuloides*), and balsam poplar (*Populus balsamifera*) hardwoods (Wilton 1965).

The resident woodland caribou prefer mature forests that have high lichen abundance. According to Brown (1986) and Schaefer et al. (1999), their main nonhuman predators are wolves and black bears (*Ursus americanus*). There are specific caribou reserves that exclude timber harvesting in current forest management plans for Labrador District 19A. With very little commercial development in the area, the people, the land, and the caribou still reflect historic patterns and interactions.

### Model overview

We used a mean-reverting process (Dixit and Pindyck 1994) to describe the expected amount and temporal variability of caribou habitat. Mean-reverting processes incorporate the volatility and speed of reversion to the mean of the system being assessed. In our application, the volatility and speed of reversion are dictated by the forest dynamics resulting from natural and human sources. The stopping rule used in the formulation of our real options problem specifies that harvesting must be shut down when the amount of caribou habitat approaches a critical minimum threshold. The timing of the decision to end harvesting reflects the social trade-off between the loss of the socioeconomic opportunity from timber harvesting and the risk of having insufficient habitat to maintain the caribou population. The timing is sensitive to the valuation by society of both the existence of the caribou herd and the benefits associated with timber exploitation; it is also sensitive to the uncertainty associated with the system. Decision makers may be more cautious if their understanding of the system is limited, for example, if they lack confidence about the extent and frequency of forest disturbance, or if they have reservations about how a caribou population will respond to timber harvesting activities.

### Ecological context

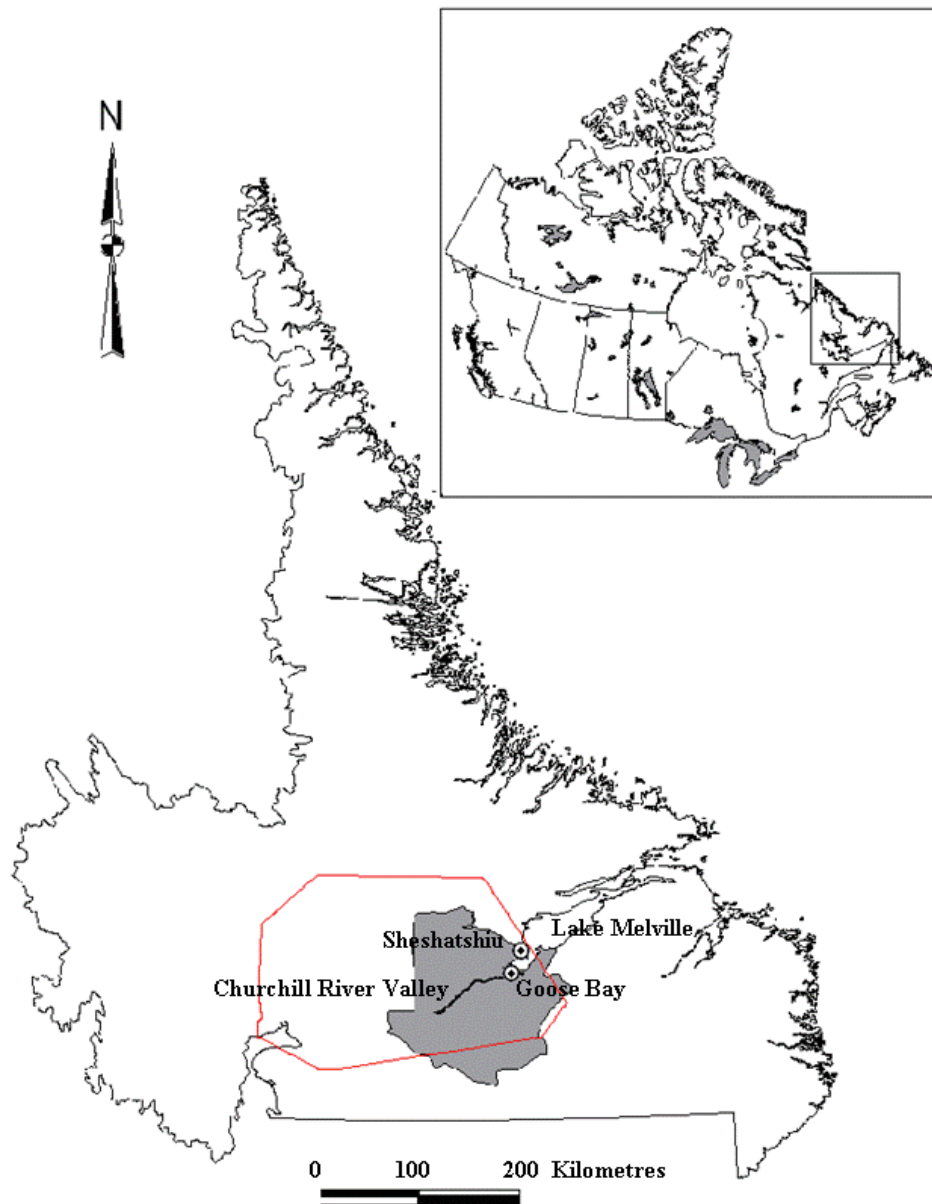
To construct the real options model, we first examined the ecological dynamics of the central Labrador study area and investigated the habitat requirements of the RWM caribou herd. To model the trade-off between caribou habitat and timber harvesting, the real options methodology needs to capture the inherent ecological and forest management processes and the ecological boundaries of the system. Tracking the amount and variability of habitat through time is central to the model. As well, the model requires some approximation of a minimum amount of habitat below which caribou survival becomes questionable.

#### *Caribou habitat*

To maintain a species, a minimum viable population is required. This is characterized as a population that can exist without facing extinction from natural disasters or demographic, environmental, or genetic stochasticity (Shaffer 1981). The model in this study focused on using the amount of habitat as a surrogate for maintaining a viable population. As commercial harvesting progresses across the landscape, the mature forest becomes increasingly fragmented; this increases caribou mortality by compromising their ability to spread out across the landscape to avoid predators and exposing them to incidental hunting. We assume that other measures, such as hunting restrictions, required to protect the caribou would be maintained. Caribou habitat typically includes large contiguous areas of old forest with terrestrial lichen, peat land, and bog complexes with a minimum of human disturbance (Seip 1998, Johnson et al. 2002, Schmelzer 2004). To capture the old forest requirement, we characterized forest stands greater than 160 yr old as woodland caribou habitat. This habitat requirement happens to be the one most directly affected by forest management.

It is widely considered that caribou require some minimum amount of habitat to survive in landscapes with commercial forestry, although the exact level is hard to identify (Seip and Cichowski 1996, Smith et al. 2000, Schaefer 2003). Attempts have been made to identify the critical amount of habitat required for a species to persist (Fahrig 2001). However, in field studies a minimum habitat threshold is often not known until it has been crossed (Carpenter et al. 2001). Given the challenges of explicitly identifying an appropriate habitat threshold, we relied on theoretical limits. The

**Fig. 1.** District 19A study area and an outline of the historic range of the Red Wine Mountains caribou herd in central Labrador, Canada.



conservation literature suggests that, in general, a landscape becomes fragmented to an organism when  $\leq 30\%$  of its habitat is intact (Andr n 1994, Fahrig 2002). We chose 30% of the expected amount of old forest as our critical threshold because of the overwhelming impact that fragmentation has on caribou population viability. We recognize that the actual minimum threshold for the RWM caribou may be different because of landscape and species differences when compared to the theoretical literature (Andr n 1999).

### *Habitat dynamics*

The expected amount of old forest, and therefore habitat, can be estimated based on the frequency and extent of forest growth and stand-replacing disturbances such as fire and timber harvesting. We used a landscape-level disturbance simulation model (Fall et al. 2004) to estimate the expected amount and variability of habitat. In overview, the simulation model captures both landscape disturbance and forest management and generates indicators of forest structure through time. It was implemented using the SELES (Spatially Explicit Landscape Event Simulator) modeling system (Fall and Fall 2001). This software is a flexible tool for building and processing grid-based spatio-temporal models. Fire was modeled based on its historic frequency and extent (B. Sturtevant, *unpublished data*), and the forestry regime was modeled based on the rate of harvest as described in current forest-management planning documents (Forsyth et al. 2003). The simulation model was then used to conduct a set of Monte Carlo simulations based on the fire and harvesting regimes, generating 100 data sets containing the amount and variability of caribou habitat through a 1000-yr period. These data were then used to generate the parameters of the real options model.

Fire is the main natural disturbance agent in the boreal forest (Johnson 1992, Payette 1992). To estimate the fire regime, we used Labrador provincial forest-fire data for the study area (B. Sturtevant, *unpublished data*). Based on the size and frequency of fires in the past 35 yr, a fire rotation, i.e., the amount of time required to burn an area equivalent to the study area, of 343 yr was calculated. This fire rotation is consistent with the longer 500-yr rotations reported for the wetter east coast of Labrador (Foster 1983), and is longer than the rotations of 123–273 yr in the drier areas to the west in central Quebec (Bergeron et al. 2001). To

aid in the parameterization of the landscape simulation model, the theoretical mean expected amount of forest older than 160 yr, i.e., our caribou habitat, can be calculated using a negative exponential distribution (Van Wagner 1978, Johnson et al. 1995). The equation has the form  $A(t) = e^{-t/b}$ , where  $t$  is the time since last disturbance in years and  $b$  is the disturbance cycle of 343 yr. Solving the equation  $A(160) = e^{-160/343} * 1,117,327$  of forested area gives the estimate of 700,800 ha of forest older than 160 yr. Given that we have defined the critical minimum habitat threshold required to maintain the RWM caribou population as 30% of the expected amount of old forest, we can calculate the minimum amount of habitat as 30% of 700,800 ha ( $H_c = 210,240$  ha).

To parameterize the simulation model, estimates of the fire cycle and number of fires per year are needed (Fall et al. 2004). The number of fires per year is equal to the forested area divided by mean fire size and fire cycle. Using the 343-yr fire cycle and an average fire size of 1003 ha calculated on the size of the fires in the study area over the past 35 yr (B. Sturtevant, *unpublished data*) would imply a mean of 3.25 fires per year. Using the expected mean number of fires and mean fire size, the simulation model selects the fire size and number of fires per simulated year from an exponential distribution (Fall et al. 2004). It was found that simulated fires did not burn as large as expected because of fire starts on islands and other types of barriers that prevented fires from reaching their full extent. To force the model to match the extent of burning predicted in theory, the simulation model was adjusted to burn more frequently by using a 295-yr return interval and a mean of 3.78 fires per year, which leads to an age class structure consistent with a 343-yr return interval.

The simulation model uses forest growth and harvesting submodels to characterize the study area's timber management regime and spatially captures timber harvesting blocks and road networks. With each simulated time step, the forest is aged and timber volume is calculated based on growth and yield projections (Newfoundland and Labrador Department of Natural Resources, *unpublished data*). Planned harvesting activities are modeled by specifying an annual volume of timber to be harvested. An "oldest first" harvest rule is used to assign priorities to forest stands for harvest.

## The real options model

### Formulation of the model

Although the current amount of habitat ( $H$ ) is known at any point in time, uncertainty exists about future projections of  $H$ . Its behavior is not totally random, because it tends toward some average long-run level that differs depending on the presence of timber harvesting. Because of the many uncertainties that affect the future amount of caribou habitat, it becomes impossible to know how quickly the long-run equilibrium level is being approached. Furthermore, the amount of habitat may fluctuate through time, such that it oscillates around an expected level.

The expected amount of old forest, and therefore habitat, was estimated using the results from the simulation model of landscape-level forest disturbance. The habitat time series generated by the Monte Carlo simulations was analyzed to determine the most appropriate stochastic process to describe habitat dynamics. Using the econometrics software EVIEWS (QMS 2007), we can show that the Neperian logarithm  $h$  of the habitat amount  $H$  ( $h = \text{Log}(H)$ ) follows approximately a mean-reverting process. The mean-reverting process is characterized by the speed of reversion ( $\alpha$ ), volatility ( $\sigma$ ) and equilibrium long-run level  $\underline{h}$ , such that

$$dh = \alpha(\underline{h} - h)dt + \sigma dz \quad (1)$$

where  $dz$  is the increment of a Wiener process, a stochastic term with a normal distribution, and  $dt$  is the change in time. Thus  $dh$  represents the change in  $h$  over a time interval  $dt$ . The component  $\alpha(\underline{h} - h)dt$  reflects the deterministic behavior of the amount of habitat or what would happen in the absence of stochasticity. The deterministic component is null when  $h$  is equal to its equilibrium long-run level ( $\underline{h} = h$ ). When  $h$  is higher than its equilibrium long-run level  $\alpha(\underline{h} - h)dt$ , it is negative; this implies a reduction in  $h$ . When  $h$  is lower than  $\underline{h}$ , the opposite is true. In both cases, the speed of adjustment is proportional to the gap  $\underline{h} - h$  and to  $\alpha$ , the speed of reversion parameter.

The stochastic component

$$\sigma dz = \sigma \varepsilon \sqrt{dt} \quad (2)$$

reflects the unpredictable natural variation because of external effects, such as fire, where  $\varepsilon \sim N(0,1)$ .

If the log of the amount of habitat is  $h_0$  at time zero, the expected amount of habitat at any time  $t$  is

$$E_0 h_t = \underline{h} + (h_0 - \underline{h})e^{-\alpha t} \quad (3)$$

and its variance is

$$\text{Var}_0 h_t = \frac{\sigma^2}{2\alpha} (1 - e^{-2\alpha t}) \quad (4)$$

Over the long run, the amount of habitat is expected to fluctuate on average around  $\underline{h}$ , a level that will differ based on the presence or absence of timber harvesting.

To address the impact of forestry on habitat, i.e., whether harvesting is allowed or banned, we assume that the forestry regime affects only the level of  $\underline{h}$ . Depending on whether timber harvesting is prohibited or not, the long-run amount of habitat  $\underline{h}$  may take two possible values,  $h_a$  or  $h_b$ . The variable  $h_a$  is the long-run amount of habitat when timber cutting is allowed, whereas  $h_b$  is the long-run amount of habitat when timber harvesting is banned. That is,

$$\nabla h \equiv h_a - h_b < 0 \quad (5)$$

represents the long-run effect of timber harvesting on the amount of caribou habitat.

Whether forestry is allowed or not, the amount of habitat may decline because of forest dynamics such

as wildfire to a critical amount  $h_c$ , at which time the caribou herd may become extinct. However, timber harvesting, by reducing the long-run equilibrium level of habitat, will cause  $h$  to be on average lower than if timber exploitation were not allowed, thereby increasing the probability that the resident caribou will become extinct. Consequently, the decision maker has to balance the benefits of allowing timber harvesting against the costs in terms of an increased probability of caribou extinction.

### Parameterization

The critical minimum habitat threshold for maintaining a viable population of caribou has been estimated based on the ecological literature as  $h_c = 12.26$  ( $H_c = 210,240$  ha). The stochastic process parameters were estimated using the Maximum Likelihood Approach (Gourieuroux and Jasiak 2001). Using this process when harvesting is banned,  $h_b = 13.45$ ,  $\alpha = 0.0532$ , and  $\sigma = 0.0529$ . Similarly, when harvesting is allowed,  $h_a = 12.70$ ,  $\alpha = 0.0575$ , and  $\sigma = 0.0533$ . We can assume that  $\alpha = 0.05$  and  $\sigma = 0.05$  and that they are almost independent of the harvesting regime.

### The objective functions

We assume that the local community is more concerned about the extinction of the RWM caribou herd than about the exact number of caribou. Caribou provide a constant instantaneous utility, denoted by  $s$ , to the community as long as they exist and *zero* if they become extinct. Further, we assign an instantaneous utility, denoted by  $\rho$ , to the sale of timber. We assume that the decision maker espouses the objectives of the community. In that case, the Net Present Value of the forest when harvesting is banned is the expected utility flow from the caribou's existence, or implied by their extinction, and is

$$V(h) = E \int_0^{+\infty} s_t e^{-rt} dt \quad (6)$$

where  $s_t$  is defined as follows

$$\begin{aligned} \text{If } \forall s \in [0, t], h_t > h_c \text{ then } s_t &= s \\ \text{If } \exists s \in [0, t] \text{ such that } h_t &= h_c \text{ then } s_t = 0 \end{aligned} \quad (7)$$

This means that the utility flow associated with the caribou herd is  $s$  as long as  $h$  remains higher than the critical habitat threshold value  $h_c$ , and becomes null forever as soon as that level is reached, even if there is a recovery in the amount of habitat. As the Net Present Value  $V(h)$  depends on the moment when the instantaneous utility  $s_t$  shifts from  $s$  to *zero*, then  $V(h)$  depends on the stochastic process followed by  $h$ , which is itself determined by the forestry policy regime. Prohibiting timber harvesting decreases the probability of caribou extinction but implies foregoing future timber revenues as long as the prohibition is effective. We assume that, once decided, the prohibition of timber harvesting is irreversible, costs nothing, and applies forever. We assume that, under the harvesting regime, the forest provides a constant revenue flow of  $\rho$  by unit of time.

We assume that timber harvesting will be allowed at the beginning of the model. Furthermore, we are investigating a forest that has not been extensively exploited before for commercial purposes, so we assume that  $h$  is initially high and that it is possible to start harvesting timber without threatening the caribou with extinction for some time into the future. However, as forestry proceeds,  $h$  will diminish according to the process discussed above. At some stage, caribou habitat may become dangerously close to the threshold  $h_c$ , and timber harvesting may have to be banned to reduce the risk that the level of habitat might fall below  $h_c$ .

While harvesting is still allowed, the decision maker has an option to prohibit harvesting. Once in place, the ban is irreversible. Let  $F$  be the value of the forest when harvesting is allowed and the caribou still exist. The value of  $F$  includes the option to prohibit harvesting once and for all, and its value stems from the utility provided by caribou existence and timber revenues.  $F$  is entirely anticipated by the decision maker based on the expected future amount of habitat according to the stochastic process governing  $h$ .  $F$  is then a function of  $h$ . This value is enhanced because of the flexibility of the decision maker to improve caribou protection by banning harvesting. The decision maker must choose a decision rule that will yield the optimal future time to prohibit harvesting. To achieve this goal, the following maximization problem needs to be solved by choosing the banning date

$$F(h) = \max_T E \left\{ e^{-rT} V(h_T) \right\} \quad (8)$$

The solution must satisfy the Value Matching (VM) and Smooth Pasting (SP) conditions

$$(VM) : F(h^*) = V(h^*) \quad (9)$$

$$(SP) : F'(h^*) = V'(h^*)$$

The decision maker decides to prohibit timber harvesting at time  $T$ , when the amount of caribou habitat first hits stopping threshold  $h^*$ . This threshold is sufficiently above the critical minimum habitat threshold  $h_c$ . In this calculation,  $h^*$  should depend negatively on  $\alpha$  but positively on  $h_a$ ,  $h_b$ , and  $h_c$ . To decide when to ban harvesting, the decision maker has to monitor the current amount of caribou habitat and ban harvesting forever as soon as  $h$  hits the critical value  $h^*$ .

A detailed resolution of the maximization equation is provided in Appendix 1.

The complete resolution consists in computing  $h^*$  once  $V(h)$  and  $F(h)$  have been determined. Decision makers will then be able to ban harvesting optimally at some time in the future as follows: They will monitor the amount of caribou habitat over time and ban timber exploitation the first time  $h$  hits the threshold value  $h^*$ . Hence, the decision maker cannot predict the time at which banning will be applied because it depends on future realizations of  $h$ . It is worth noting that banning may never happen if over time  $h$  diverges from  $h^*$ .

## RESULTS

The real options model was applied to the Labrador study area by computing numerically  $V(h)$ ,  $F(h)$ , and the stopping threshold  $h^*$ . The real options analysis involves generating a series of model runs that are then interpreted by a forest manager. The runs vary, as reflected in the value of current habitat  $h$ , because of the stochastic nature of the model, which captures the combined dynamics of the forest management regime and natural disturbance. A decision maker evaluates model output and gains

insights into how the amount of caribou habitat fluctuates over time according to various random future events.

To illustrate the real options methodology, we present three examples of our real options model output. These examples show the importance and sensitivity of the critical threshold  $h_c$ . The model output balances the benefit from continuing harvesting while keeping caribou under an acceptable, but perhaps higher, probability of extinction on the one hand with the economic loss from prohibiting harvesting to reduce this probability on the other. Table 1 shows the parameters used in the real options model for the Labrador study area. We have assigned an instantaneous value from timber ( $\rho$ ) to 1, and the instantaneous value of caribou ( $s$ ) has been assigned 2 to demonstrate the real options methodology. The values of  $s$  and  $\rho$  reflect how risk averse the Labrador decision maker is: A higher value of  $s$  will stop harvesting sooner, and lesser values will stop timber extraction later. Finally, the interest rate is taken to be equal to 5%. The numerical resolution of the model leads to  $h^* = 12.2670$ , which reflects the Net Present Value of the instantaneous utility value of  $s$  and  $\rho$ .

Figures 2 through 4 illustrate three possible realizations of the habitat amount over a period of 1000 yr. Each realization is obtained by generating  $h$  using the differential equation of  $dh$  and starting from the current habitat amount ( $h_0$ ). Among many possible realizations generated this way, we have chosen examples that demonstrate how the model works and how decisions can be made.

As previously discussed and depending on the future realization of  $h$  starting from its present value of  $h_0 > h^*$ , harvesting is irreversibly banned as soon as  $h$  hits  $h^*$ . The examples (Figs. 2 to 4) illustrate how the time to ban harvesting is a stochastic variable taking value for the interval  $[0, +\infty]$ . Initially, harvesting is allowed, and the current value of  $h$  is  $h_0$ . This is sufficiently larger than  $h_c$  to justify harvesting, at least for a while.

The first graph (Fig. 2) illustrates an outcome that maintains caribou habitat above  $h^*$  and thus allows for timber harvesting. Figure 2 shows example 1 of the variation in the amount of caribou habitat ( $h$ ) over 1000 yr according to the stochastic mean reverting process

**Table 1.** Descriptions and values of parameters for the real options model.

| Parameter description   | Value              | Lognormal value |
|---|--------------------|-----------------|
| Amount of habitat at time 0   | $H_0 = 467,185$ ha | $h_0 = 13.0545$ |
| Critical minimum habitat threshold  | $H_c = 210,240$ ha | $h_c = 12.2560$ |
| Speed of reversion of the mean-reverting function   | $\alpha = 0.05$    | ...             |
| Volatility of the mean-reverting function   | $\sigma = 0.05$    | ...             |
| Interest rate   | $r = 5\%$          | ...             |
| Instantaneous utility from timber or the flow per unit of time of revenue from selling timber                     | $\rho = 1$         | ...             |
| Instantaneous utility from caribou or the flow per unit time of (equivalent) revenue from the caribou's existence | $s = 2$            | ...             |

$$dh = \alpha (h_a - h) dt + \sigma dz \quad (10)$$

where  $h_a$  is the equilibrium long-term amount of habitat when harvesting is allowed, and the natural landscape dynamics are captured by the speed ( $\alpha$ ) and volatility ( $\sigma$ ) of reversion to the long-term amount of habitat. Under this realization of  $h$ , the amount of habitat stays above the critical stopping threshold  $h^*$  and timber harvesting is allowed for at least 1000 yr.

In the second future realization of  $h$  (Fig. 3),  $h$  diminishes to  $h^*$  in approximately 300 yr. This implies that harvesting must be prohibited at that time because it is getting close to the critical minimum habitat threshold  $h_c$ . Prohibiting harvesting when  $h$  hits  $h^*$  will let  $h$  be governed by a new stochastic differential equation, Eq. 11, instead of Eq. 10:

$$dh = \alpha (h_b - h) dt + \sigma dz \quad (11)$$

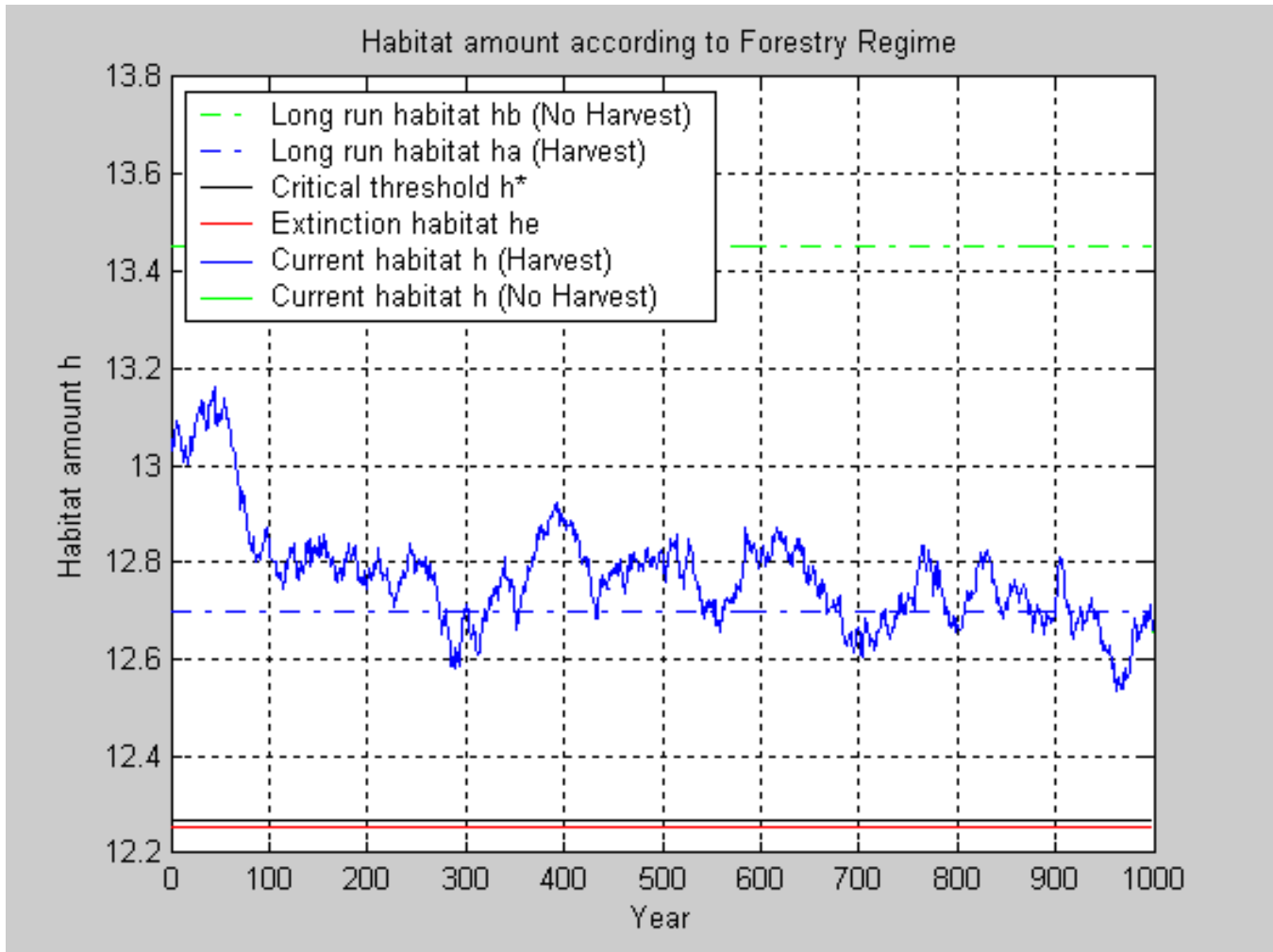
As  $h_b > h_a$ , prohibiting harvesting will give  $h$  a better chance to rise and remain higher than  $h_c$ . This does not guarantee that the caribou herd will not become extinct over the long run but makes that outcome less probable.

The third possible future realization of  $h$  (Fig. 4) shows that prohibiting harvesting after almost 800 yr does not succeed in keeping  $h$  above  $h_c$ , because  $h$  follows the mean-reverting process in Eq. 11 and the amount of habitat declines to the critical minimum habitat amount threshold ( $h_c$ ), at which time the caribou herd hypothetically goes extinct because of natural landscape dynamics despite the fact that harvesting has been banned. In the graph, the habitat amount is maintained equal to  $h_c$  after the caribou have gone extinct to underline the fact that extinction is an irreversible event.

## DISCUSSION

The real options model that we have described focuses on uncertainty and irreversibility in a dynamic context. The irreversibility in the model applies to events such as extinction or decisions such as banning timber harvesting, and the uncertainty pertains to how the amount of caribou habitat will change, because the current level of habitat is observable but its future level is unknown. The model specifies that the objective of the decision maker is to maximize future benefits from the forest, given the uncertain benefits derived from caribou and from timber exploitation. For this model, the sole instrument that the decision maker can use to achieve the objective of maximizing future benefits is to ban timber harvesting forever, a decision to be

**Fig. 2.** Example 1 of the variation in the amount of caribou habitat ( $h$ ) over 1000 yr, following the stochastic mean reverting process in which the amount of habitat stays above the critical stopping threshold  $h^*$  and timber harvesting is allowed for at least 1000 yr.

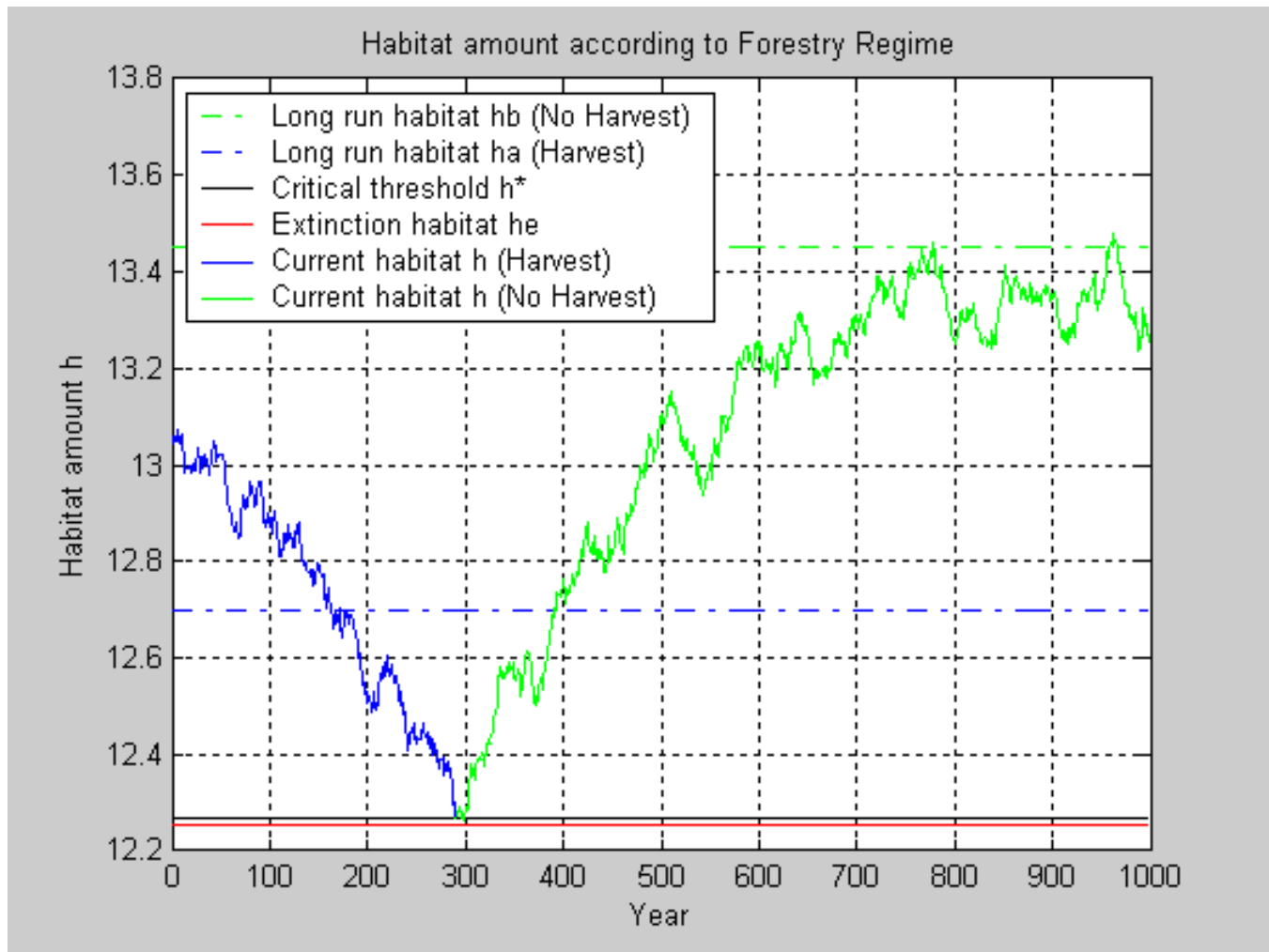


taken at some future time. Given the uncertainty surrounding the future, the decision maker would be mistaken to specify a definite future date at which harvesting should be banned, because the habitat level might be more than adequate at that date, so that no ban would be needed, or it might become dangerously low before that date, in which case the ban should be introduced earlier. Consequently, the decision maker should not choose a date but a decision rule to be applied at all future dates. That rule consists of observing future levels of habitat and banning timber exploitation the first time the

threshold value  $H^*$  is reached. Although the date at which this may happen is uncertain, the threshold itself is not random. It is computed according to the various ecological, biological, and economical parameters of the model and according to the stochasticity of the process governing  $H$ .

Analyses of real options models indicate some interesting properties of the decision rule and the threshold value. It can be shown that, by applying the stopping rule, decision makers attempt to provide their communities with favorable

**Fig. 3.** Example 2 of the variation in the amount of caribou habitat ( $h$ ) over 1000 yr. For the first 300 yr, the stochastic mean reverting process that allows harvesting is used to determine the supply of habitat. After 300 yr, the critical stopping threshold  $h^*$  is reached, harvesting stops, and the variation in the amount of habitat then follows the stochastic mean reverting process, which bans timber harvesting.

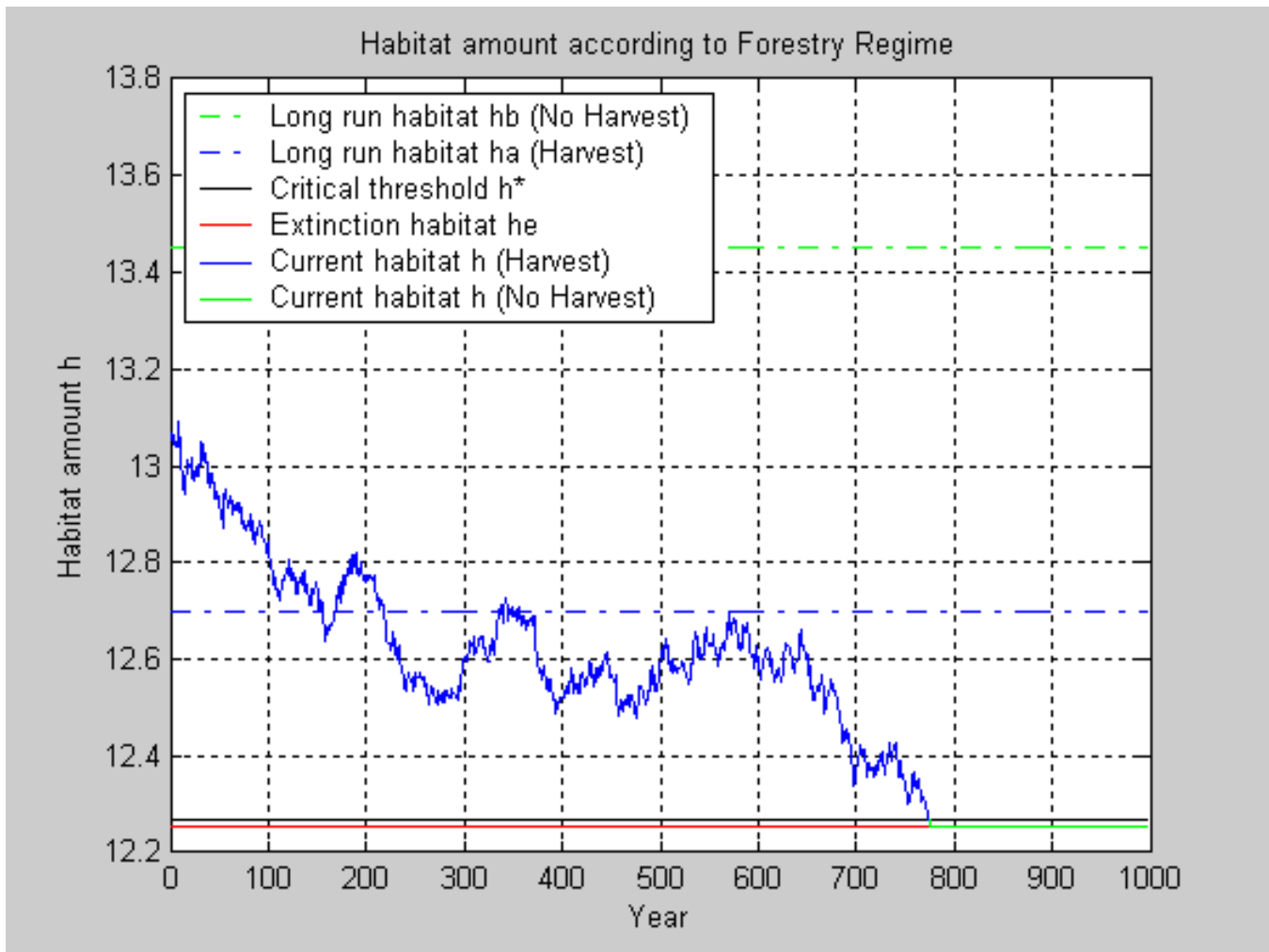


outcomes, or cases in which  $H$  grows more than expected, while protecting them from unfavorable outcomes or low growth and thus maximizing future benefit. In practice, this is achieved by banning harvesting when  $H$  is still sufficiently above the critical minimum habitat threshold. This does not guarantee that the caribou will not be extirpated from the study area, but will increase their chance of survival. In that respect, the model is a rigorous application of the precautionary principle. It does not prohibit risk taking, but agrees with the intuitive

conventional wisdom that decisions should bend the distribution of risk a community is exposed to in a way that reduces the probability of irreversible catastrophes and thereby maximizes the future benefits of an active forest industry and the existence of woodland caribou.

One key parameter in the model is the critical minimum habitat threshold ( $H_c$ ) below which the caribou, according to the underlying biological theory, becomes extinct. This value is exogenous to

**Fig. 4.** Example 3 of the variation in the amount of caribou habitat ( $h$ ) over 1000 yr. For the first 800 yr, the stochastic mean reverting process that allows harvesting is used to determine the supply of habitat. After 800 yr, the critical stopping threshold  $h^*$  is reached, harvesting stops, and the variation in the amount of habitat then follows the stochastic mean reverting process, which bans timber harvesting. Despite the harvesting ban, the amount of caribou habitat falls below the critical minimum habitat threshold because of natural landscape dynamics. The amount of habitat is then maintained equal to  $h_e$ , reflecting the hypothetical extinction of the caribou, an irreversible event.



the model in that it is determined by ecologists. Although certain according to the theory, it is not known accurately in practice. Underestimating  $H_e$  could lead to accidental extinction because the model would wait too long before prohibiting harvesting. Too high an estimate may appear wise but implies foregoing timber revenues unnecessarily. In fact, the choice of the stopping threshold ( $H^*$ ) as

determined by the model incorporates the precautionary principle, but only to the extent that risk arises from uncertainty in the evolution of  $H$ . The scientific risk of an error in  $H_e$  is not taken into account.

In addition, socioeconomic issues can and must be addressed. It is, at least conceptually, easy to

evaluate the benefits from timber harvesting leading to parameter  $\rho$ , the economic revenue from timber extraction, and it is certainly useful to do sensitivity analyses around it. More difficult is the issue of the value associated with caribou existence ( $s$ ). Some will argue that caribou cannot be valued. Does this mean that they are valueless, in which case the forest has no value when harvesting is banned and the Net Present Value is 0 ( $V(h) = 0$ )? Does it mean that their value is infinite? A positive answer to either of these questions implies that there is no timber harvesting issue: Allow harvesting forever in the first instance, and prohibit it forever in the second instance. Thus the harvesting decision is not a trivial one, and the caribou must have a finite value. The model does not determine that value, but it can help investigate the consequences of that value in terms of allowing or banning timber harvesting. Given the complexity of the problem, what is required is fundamentally a social decision on the value of the caribou herd and the value of timber harvesting to balance the uncertainties, risks, and irreversibility issues involved in the forestry-caribou conflict.

The real options methodology provides several advantages over traditional approaches to managing forestry-wildlife conflicts such as the one in our Labrador case study. Unlike conventional forest management approaches, which depend on the certainty and consistency of the future supply of the resource (Gunderson 2000), the uncertainty about the complex dynamics of natural systems is central to the real options decision making process. The real options approach does not provide a deterministic solution as to when timber harvesting should be stopped, but instead provides a decision rule and process that allow future decisions to take new information into account as it arises. By adapting to new developments, this process has the potential to maximize the future supply, and therefore the benefit, of socially valued ecosystem services such as timber and wildlife. A decision maker is not presumed to have complete knowledge of the current system and its future, but only the capacity to respond to change.

Implementing forest dynamics numerically requires a simplification using only a mean and deviation. More complex behaviors, such as the longer-term oscillations resulting from climate cycling, are far more challenging to include. Similarly, it is a challenge to include in this numerical method the lag effects of changes in habitat on wildlife populations. In this application, a simple threshold

was used, and caribou population dynamics were not incorporated explicitly. As well, caribou habitat was characterized simplistically, as forests greater than 160 yr old, and other aspects of caribou habitat, such as habitat connectivity, were not included.

Some improvements and extensions might be considered in further work. For example, the ban on timber harvesting might be reversible. In such a case, the decision maker would consider two threshold levels: one lower level at which harvesting would be temporarily banned, and one higher level at which harvesting would be reinstated temporarily if  $H$  recovered sufficiently. Finally, there may be actions and decisions other than timber harvesting that affect caribou populations and survival probabilities. These should ideally be considered in the timber harvesting vs. caribou preservation debate. However, as explained in this paper, it is when some degree of irreversibility combines with uncertainty that the real options approach is most useful and telling.

Responses to this article can be read online at:  
<http://www.ecologyandsociety.org/vol13/iss1/art27/responses/>

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### Acknowledgments:

*This project was funded through the Sustainable Forest Management Network of Canada, the University of Québec at Montreal, the Social Science Research Council of Canada, and the British Columbia Ministry of Forests and Range. We would also like to thank Andrew Fall, Neal Simon, Doug Steventon, Brian Sturtevant, and two anonymous reviewers for valuable comments on the manuscript.*

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**Appendix 1.** Resolution of  $V(h)$  and  $F(h)$ .

*[Please click here to download file 'appendix1.pdf'.](#)*

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