

APPENDIX 2 – Modified Cobb-Douglas Utility Function

In this paper we used a modified multiplicative Cobb-Douglas function to represent the resident location decision-making process. The utility function is used in a boundedly rational decision-making context, whereby the resident selects the optimum location based on a constrained number of location options. Various functions (e.g., hedonic and logit) and decision-making strategies (e.g., optimization, belief-desire intentions) have been used to represent agent decision-making behavior and have been the focus of much work in the social sciences. We have not tested the psychological validity of the multiplicative Cobb-Douglas utility function used in this research. However, previous work by Rand et al. (2003) demonstrated that the decision-making of agents using this function in SOME can generate distributions of cluster sizes that compare well with the structural form of real-world cities.

In addition to its various analytical advantages (see Chiang 1984), we selected the multiplicative Cobb-Douglas function over the additive because it eliminates the possibility that a location with zero suitability on one factor will have a non-zero utility. As illustrated in the two-factor example shown in Figure A2.1, iso-utility curves for the multiplicative function have a negative slope and are asymptotic to zero when one factor has a very high score (and the other approaches zero). The convex shape of the curve emphasizes the importance of the trade-off among all preference factors. In contrast, the additive function allows locations with zero suitability on one factor to have non-zero utility and, thus, does not represent the trade-offs as well.

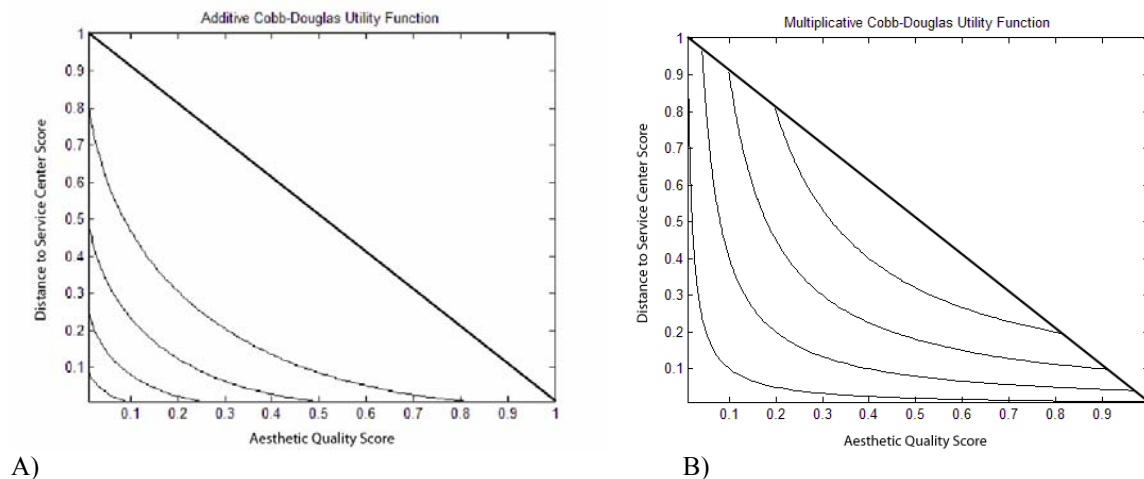


Figure A2.1: The above graphs plot the iso-utility curves for a two-factor case using (a) the additive form and (b) the multiplicative form. The iso-utility curves represent combinations of preference scores for which the utility remains the same, given constant and equal preference weights ($\alpha=0.5$). The continuous surface is shown using curves for utility = 0.4, 0.6, 0.8, and 1.0 in (a) and utility = 0.1, 0.2, 0.3, and 0.4 in (b). The dark diagonal line represents our constraint that the factor scores are in the range zero to one.

Typically a Cobb-Douglas utility function is linearly homogenous (Chiang 1984), meaning that, for example, doubling the input factor scores leads to a doubling of the utility and the trade-offs

between the factors remain the same. These constant returns to scale are the result of constraining the factor weights to sum to 1; i.e., a primary assumption of the Cobb-Douglas function is that given the factor weight α is that the second factor weight is $1 - \alpha$, in a two factor equation. (Cobb and Douglas 1928). Therefore to maintain the Cobb and Douglas (1928) assumptions, given n factors (i.e., three) in our utility function, at most $n - 1$ (i.e., two) weights can be drawn independently. It was necessary to constrain our preference weights to sum to one because it normalizes the utility function and is the only method that allows for inter-agent comparison of well-being (Elster and Roemer 1991) and analysis of disparity in satisfaction levels within the population. In the absence of absolute measurements (e.g., I receive 500 units of satisfaction from being 10 km from work), relative measurements of utility or satisfaction are better and more useful than welfare economic approaches using Pareto efficiency (Elster and Roemer 1991). The sum-to-one constraint eliminates complete independence of preference-weight assignment.